Problems with a small parameter are omnipresent in nature. Think, for example, of the Hodgkin-Huxley model describing how action potentials in neurons are initiated and propagated. Often these small parameters have little influence on the solutions of the model and a good approximation can be obtained by neglecting the perturbation. We call these problems regularly perturbed. This is in contrast to singularly perturbed problems for which the small parameters have a huge effect on the solutions and incomplete or even utterly wrong approximation will be obtained by simply neglecting the small terms. We will discuss the effects a small parameter can have on singularly perturbed (differential) equations and how to still obtain good approximations for the solutions from the reduced problems. To do so, we introduce Geometrical Singular Perturbation Theory and Fenichel Theory. In the end, we will apply these methods to study a model used to describe gas-discarge on a phenomenological level. This is a model which is very close to my heart and which I have been studying ever since I started my PhD in 2005.

References

1. M.H. Holmes: Introduction to Perturbation Methods
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